Modeling and optimizing risk in the strategic gas-purchase planning problem of local distribution companies

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The liberalization of the European energy market has enabled big gas customers and public utilities to build a portfolio of different gas supplies and purchase contracts. The covering of the gas demand, which is heavily temperature dependent, can be optimized by combining baseload contracts, open gas delivery contracts, and the use of the capacity of underground and local pipe storage facilities. We present a two-stage stochastic linear programming model for the optimization of the gas-purchase portfolio under uncertain demand conditions while considering the cost of purchase, underground storage capacities and transportation. Furthermore, we enhance the model to explicitly consider conditional value-at-risk. We evaluate our approach based on a real-world case study. The results show that our model is computationally tractable by a standard interior point solver for hundreds of scenarios. It clearly outperforms alternative deterministic planning approaches such as scenario analysis both in terms of expected profit and robustness.

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1 INTRODUCTION

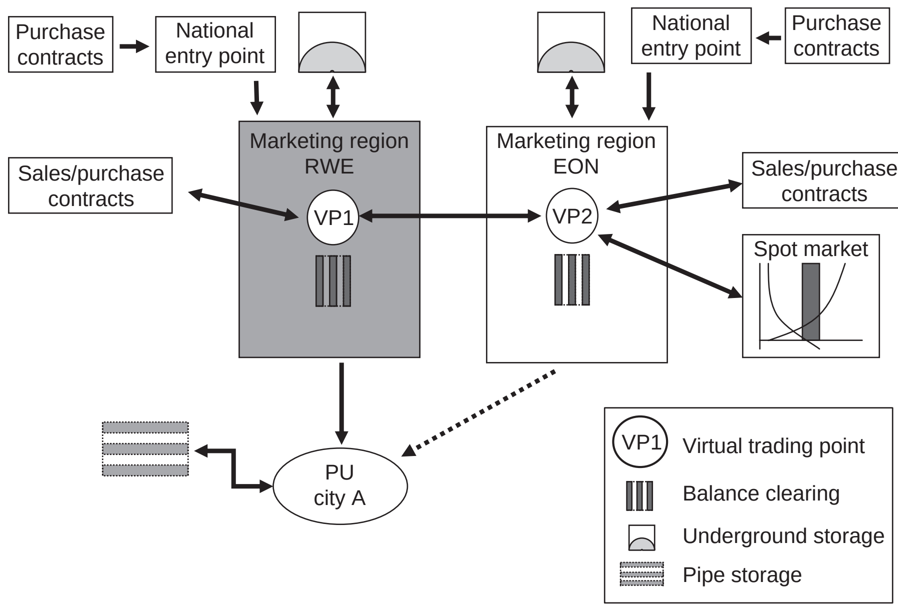
In the last fifteen years, natural gas has become one of Germany's major sources of energy, with a share of 23% of the total primary energy consumption in 2007 (see Eurogas (2007)). This share is expected to grow to 30% by 2030, while the absolute consumption of natural gas is expected to grow by the same amount, mainly due to increasing gas-based electricity production (see Lanhenke *et al* (2007)). At the same time, the liberalization of the European gas market commenced in 1998 with directive 98/30/EG of the European Parliament and European Commission and was implemented in Germany with the Energy Industrial Act of 2005 (Deutscher Bundestag (2005)). Germany's gas market is characterized by a complex, historically grown three-tier gas value chain of a few importing companies, about fifteen regional transmission system operators and more than 700 local distributors, mostly public utilities (see Scheib *et al* (2006) and Deutsche Bundesregierung (2007)).

Public utilities typically purchase gas from the importing companies and, to a much smaller extent, from the spot market, and sell it to their local customers. The purchase price depends heavily on the type and flexibility of the purchase contracts. Substantial discounts on the market price are granted if the buyer commits himself to take a predetermined amount of gas over a longer period (several months or years). Public utilities usually guarantee to cover their customers' complete future gas demand, which is highly dependent on outside temperature. Since 2008 it has been possible for the operators of the local transmission networks to balance short-term fluctuations (Aretz and Niehörster (2008)). To counterbalance long-term seasonal variations in gas demand they can lease storage capacity in underground storage facilities. These storage facilities can also be used to optimize the gas-purchase portfolio by cheaply buying and storing in summer and supplying from the stored gas in winter. The decisions about the portfolio of medium-term and long-term purchase contracts and storage capacities are strategic decisions that are usually taken on a yearly basis. The goal is to minimize costs of purchase, storage and transportation while ensuring that the uncertain demand can always be met.

Despite the high practical relevance, there are only a limited number of contributions to the research literature that focus on the purchase planning problem of local distribution companies. O'Neill *et al* (1979) present a network-flow model taking into account mass conservation and pressure constraints within a pipeline network. Uncertainty of demand is not considered. Avery *et al* (1992), Bopp *et al* (1996) and Butler and Dyer (1999) develop network-flow type, multiperiod linear programming models that represent the physical gas transportation network as a directed graph with source and sink nodes for purchase and supply contracts and storage facilities. The computational burden of these models strongly depends on the time granularity. We propose different time aggregation schemes to find a good compromise between computational burden and the validity of the model's solution. Uncertainty of input parameters, especially gas demand and price value, is found to have a major influence on the planners decisions. Scenario approaches and stochastic versions of the models are proposed, but could not be solved for a significant number of scenarios due to the exploding computational complexity and limited solver capabilities. Guldmann (1983) presents a chance-constrained approach for supply, storage and service reliability decisions under uncertain demand. This was extended by Guldmann (1986) to consider several suppliers with different contract characteristics, but, in this case, only within a deterministic framework. Guldmann and Wang (1999) propose a mixed-integer programming model and an alternative simulation/optimization-type approach to solve a simplified supply mix portfolio problem under uncertain demand, ignoring transportation and storage services. Recently, Aouam *et al* (2010) presented an analysis of stochastic programming-based hedging strategies and naive hedging strategies for the natural gas procurement problem. A variety of nonfinancial and financial purchase contracts including futures and options as well as storage contacts are considered under price uncertainty, while demand uncertainty is ignored. Furthermore, the proposed models do not account for an underlying transportation network and the associated capacity decisions and costs. Other publications model the strategic supply and transport planning of natural gas on a European scale (Perner (2002); Seeliger (2006); and Lochner *et al* (2007)). The authors present deterministic linear and mixed-integer network-flow models, with a time horizon of ten to fifteen years, that focus on the evaluation of the gas transmission infrastructure such as international gas pipelines, storage facilities and import terminals. These models are not directly applicable to the purchase planning problem of local distribution companies. Also, uncertainty of gas demand and purchase prices is not explicitly considered.

In the last two decades, tremendous progress has been made in the field of computational linear programming (see, for example, Bixby (2002)). Hyper-sparse simplex codes (see Koberstein (2008)), highly advanced implementations of interior point methods (see Meszaros (1999) and Meszaros (1997)) and multicore 64-bit hardware architectures allows us to solve linear programming problems that are several orders of magnitude greater in size. At the same time, new solution algorithms, implementations and modeling tools for stochastic programming problems have been developed (see Zverovich *et al* (2009) and Valente *et al* (2009)). Overall, these achievements allow us to solve stochastic programming problems of much greater complexity and easier integration into real-world decision support systems. In the field of financial portfolio planning, conditional value-at-risk (CVaR) was proposed by Rockafellar and Uryasev (2002) as an effective measure of risk associated with a planning decision. A linear formulation of CVaR and an efficient cutting-plane algorithm were given by Fábián (2008).

FIGURE 1 German natural gas market model.



König *et al* (2007) describe the strategic and operational planning task of local distribution companies, a basic deterministic model and our decision support system SAPHIR. In this paper, we present a two-stage stochastic version of the model. Furthermore, we transfer risk-averse decision methods from financial portfolio planning to gas-purchase planning. We show how the notion of risk can be considered explicitly by imposing a lower limit on the CVaR associated with a purchase portfolio. Alternatively, the CVaR can be considered as an optimization goal for a given target profit value.

We start with a problem description in Section 2, followed by the description of a two-stage stochastic model in Section 3. In a case study in Section 4 we evaluate the computational tractability of our model and analyse the robustness of the generated purchase portfolios compared with deterministic planning approaches. The paper ends with some conclusions and an outlook on further research in Section 5.

2 THE STRATEGIC GAS-PURCHASE PROBLEM OF LOCAL DISTRIBUTION COMPANIES IN GERMANY

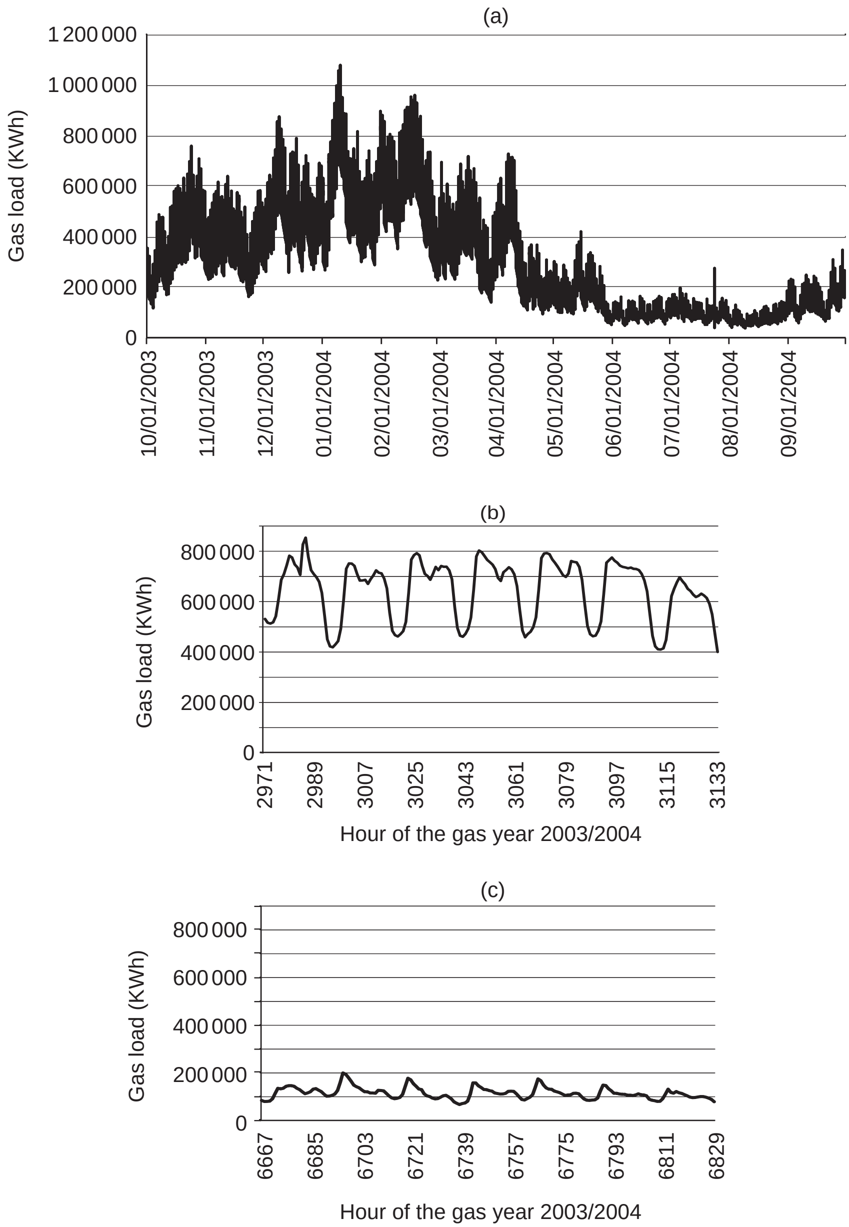
With the Energy Industrial Act of 2005 (Deutscher Bundestag (2005)), a market model for trading natural gas was established in Germany, which, to a large extent, separates the trading from the physical gas transportation. Figure 1 on the facing page depicts the market model with its typical components. The German gas market has been divided into different market regions, in which different transportation networks are integrated. For the purposes of trading, each market region is considered as a virtual trading point (VP), at which gas can be purchased, sold and distributed to other VPs. Also, gas can be purchased directly at the national entry points (such as Emden or Waidhaus). Transportation fees are billed on an annual basis at entry and exit points of a market region. They depend on the maximum amount of transportation power at entry and exit points during the year.

A public utility (PU) usually operates only on that part of the market model which is relevant for its gas purchase and the supply of its customers. The sales contracts usually commit the PUs to satisfy their customers' uncertain gas demand on an unlimited basis. Sometimes larger commercial or industrial customers accept interruptible contracts at lower prices which allow the PU to interrupt the gas supply at short notice. Prices in sales contracts usually refer to the delivered amount of energy and are fixed *a priori* for the coming year. Figure 2 on the next page shows a typical demand curve over one gas planning year, which traditionally starts in October. As the gas demand is highly dependent on temperature, it shows the typical characteristics of a yearly temperature profile: a seasonal fluctuation with high demand in winter and low demand in summer and a daily fluctuation with peak demand in the mornings and low demand at night. As peak demand has a high impact on the incurred cost, it is crucial to consider this appropriately in any kind of decision model.

Typical types of purchase contracts include yearly baseload, monthly baseload and open contracts. Baseload contracts are take-or-pay contracts. The PU purchases a fixed quantity of gas at a fixed price for a whole year or each month during the contract period, which is received at a constant power level. Purchase costs are proportional to the amount of energy and have to be paid even if the gas cannot be taken due to low demand and lack of storage capacity. In contrast to baseload contracts, the amount of energy and the power level purchased from open contracts can vary within certain intervals that have to be fixed at the beginning of the contract period. Purchase costs of open contracts are composed of two components: one depends on the purchased amount of energy, while the other depends on the maximum power purchased during the contract period. Purchase prices in open contracts are usually much higher than those of baseload contracts due to higher flexibility. Baseload and open contracts are typically closed at the beginning of each gas year and cannot be altered during the year.

Storage can be used to balance seasonal and daily fluctuations, for example, leveraging the cheap use of baseload contracts in summer and avoiding the expensive usage of open contracts in winter. Besides a fixed charge, the usage costs depend

FIGURE 2 Gas demand curves.



Part (a) shows a typical gas demand curve over one planning year starting October 1. Parts (b) and (c) depict a typical gas demand curve for a week in winter and a week in summer, respectively.

on the injection and extraction power and the total storage volume used. Additionally, compression costs are incurred for injection. Storage underlies certain physical and technical restrictions. Injection and extraction capacities heavily depend on the fill level of the storage in a nonlinear way. Furthermore, switching between injection and extraction cannot be carried out abruptly; there has to be a down time of several hours in between. For the strategic planning task, these restrictions can be approximated (nonlinear injection/extraction capacity) or disregarded (down times). If decision models are to be used for short-term planning, they have to be considered in greater detail, which inevitably leads to much greater computational complexity. In this paper, we focus on the strategic planning task, where down times can be neglected and a linear approximation can be used for determining sensible maximum extraction and injection limits.

The first German spot market for gas was opened recently at the European Energy Exchange (EEX) in Leipzig. Gas is traded as daily baseload contracts, where prices are fixed for the next day. Prices can be assumed to be largely temperature dependent in the short term and oil-price dependent in the long term.

3 A TWO-STAGE STOCHASTIC LINEAR PROGRAMMING MODEL

As in the model of Avery *et al* (1992) and its variants, which are based on the physical transportation network, the new German market model can be represented by a multi-period network-flow model. Below, we present a two-stage stochastic version of such a model, which accounts for uncertainty in gas demand by maximizing the expected profit over a certain number of discrete load scenarios. In principle, gas prices can be uncertain as well. However, they are largely known for a time period of about nine months in advance as they depend heavily on the oil price. Only at the spot market for gas do prices show significant short-term variability.

Natural first-stage decisions are the gas quantities purchased via baseload contracts, as they have to be fixed at the beginning of the gas year. Maximum storage and transportation capacities and upper limits on open contracts could also be treated as first-stage decisions. For the sake of simplicity, we do not do so in the model presented below. If baseload quantities are fixed, fluctuations in demand are handled by adjusting purchase from open contracts, transportation, and storage injection and extraction accordingly. Therefore, these decisions are determined at the second stage of the model.

Let T be the set of time periods and R be the set of demand scenarios. In the following, we use the subscript $r \in R$ to make stage-two decision variables scenario dependent. The structure of the market model is represented by an undirected graph consisting of nodes N for market regions and national entry points, and transportation links L. A quantity of gas can be transmitted from a node i to a node j in time period

$t \in T$ by setting up a power level of $\mathrm{p}^{L}_{i,j,t,r}$ megawatts (MW) on link $(i, j) \in L$. Denoting by Δ^{T}_{t} the length of a time period in hours, the amount of energy transmitted in one time period equals $\Delta^{T}_{t}\,\mathrm{p}^{L}_{i,j,t,r}$ megawatt hours (MWh). In our model, the sign of a power level indicates the direction of transmission. Power levels have to be balanced in each node of the network. The costs of transportation on a link depend on the maximum link capacity utilized in one time period of the gas year. For each link (i, j) maximum power levels $\mathrm{pmax}^{L}_{i,j,r}$ and $\mathrm{pmin}^{L}_{i,j,r}$ are determined for both directions of flow and priced with cost coefficients $\mathrm{CENTRY}_{i,j}$ and $\mathrm{CEXIT}_{i,j}$, respectively. Note that $\mathrm{pmin}^{L}_{i,j,r}$ always takes a nonpositive value.

Storage facilities are represented by a set S and can be associated with nodes i via subsets $S^{N}_{i} \subseteq S$. The fill level of a storage s in time period t is measured in terms of MWh and denoted by $\mathrm{e}^{S}_{s,t,r}$. Gas can be injected into and extracted from a storage s at power levels $\mathrm{p}^{\mathrm{INJ}}_{s,t,r}$ and $\mathrm{p}^{\mathrm{EXT}}_{s,t,r}$, respectively. As mentioned before, in storage contracts the maximum extraction and injection power levels sometimes depend piecewise linearly on the fill level. For the purpose of strategic planning these limits are approximated by time-dependent constants $\mathrm{PMAX}^{\mathrm{EXT}}_{s,t}$ and $\mathrm{PMAX}^{\mathrm{INJ}}_{s,t}$, respectively. Storage fixed costs SCF_{s} are considered as constant components in the objective function. Local distribution companies have typically only very few different storage facilities in place for potential use. We assume that the decision of which storage facility to use can be made by evaluating different network configurations in subsequent optimization runs. Alternatively, binary variables could be associated with the fixed storage costs, which would, however, increase the computational complexity of the model. Cost coefficients SCV_{s}, $\mathrm{PPRICE}^{\mathrm{INJ}}_{s}$ and $\mathrm{PPRICE}^{\mathrm{EXT}}_{s}$ account for the maximumly used fill, injection and extraction capacities and are associated with the maximum fill, injection and extraction levels $\mathrm{emax}^{S}_{s,r}$, $\mathrm{pmax}^{\mathrm{INJ}}_{s,r}$ and $\mathrm{pmax}^{\mathrm{EXT}}_{s,r}$, respectively. Additional compression costs SCC_{s} are incurred for each unit of injected energy.

Gas supply contracts are represented by the set SC and node-, time- and scenario-dependent demand $\mathrm{LOAD}_{c,i,t,r}$. As shortfalls are usually not permitted, revenues follow as constants directly from sales prices EPRICE^{SC}_{c}. In order to reflect the viewpoint of the planner, we still prefer to state the problem as a maximization problem.

The set BC contains monthly baseload purchase contracts. Sets M^{BC}_{b} indicate in which months a baseload contract b is valid. Since baseload contracts are take-or-pay contracts, we distinguish between purchase and actual utilization. While the amount of purchased gas in a month m, denoted by the decision variable $\mathrm{p}^{BC\,p}_{b,m}$, is a first-stage decision, the utilization $\mathrm{p}^{BC\,u}_{b,m,r}$ can be adapted in each scenario r. The purchased amounts may not exceed upper limits in terms of power, $\mathrm{PMAX}^{BC}_{b,m}$, and energy, $\mathrm{EMAX}^{BC}_{b,m}$. Purchase costs for baseload contracts are incurred on the basis of a purchase price per unit of energy $\mathrm{EPRICE}^{BC}_{b,m}$.

Open contracts are represented by the set OC. Purchase from an open contract o in time period t is modeled by second-stage decision variables $\mathrm{p}^{OC}_{o,t,r}$. Bounds on these variables are given by limits on the purchased amount of energy and power levels. Purchase costs of open contracts are induced by a base price $\mathrm{EPRICE}^{OC}_{o,t}$ on the purchased amount of energy per time period and a peak price PPRICE^{OC}_{o} on the maximum power level purchased during one gas year. Typically, the total purchase costs are heavily influenced by the peak component. Sometimes, purchase contracts come with threshold values for a minimum gas consumption or certain quantity discount schemes. Our model can easily be extended to consider both of these features, which would, however, require the use of binary variables and increase the computational complexity. We leave it to future research to determine whether the extended model remains computationally tractable. We also experimented with including the spot market by modeling it largely as a storage facility. Thus:

$$\max -\sum_{b\in BC}\sum_{m\in M_b^{BC}} \mathrm{EPRICE}^{BC}_{b,m}\,\Delta^M_m\,\mathrm{p}^{BC\,p}_{b,m} \tag{3.1}$$

$$+\sum_{r\in R}\rho_r\Bigg[\sum_{t\in T}\sum_{i\in N}\sum_{c\in SC_i^N} \mathrm{EPRICE}^{SC}_{c}\,\Delta^T_t\,\mathrm{LOAD}_{c,i,t,r} \tag{3.2}$$

$$-\sum_{o\in OC}\Bigg(\sum_{t\in T_o^{OC}} \mathrm{EPRICE}^{OC}_{o,t}\,\Delta^T_t\,\mathrm{p}^{OC}_{o,t,r} + \mathrm{PPRICE}^{OC}_{o}\,\mathrm{pmax}^{OC}_{o,r}\Bigg) \tag{3.3}$$

$$-\sum_{(i,j)\in L}(\mathrm{CENTRY}_{i,j}\,\mathrm{pmax}^{L}_{i,j,r} - \mathrm{CEXIT}_{i,j}\,\mathrm{pmin}^{L}_{i,j,r}) \tag{3.4}$$

$$-\sum_{s\in S}\Bigg(\mathrm{SCF}_s + \mathrm{SCV}_s\,\mathrm{emax}^{S}_{s,r} + \mathrm{PPRICE}^{\mathrm{INJ}}_{s,r}\,\mathrm{pmax}^{\mathrm{INJ}}_{s,r} + \mathrm{PPRICE}^{\mathrm{EXT}}_{s}\,\mathrm{pmax}^{\mathrm{EXT}}_{s,r} + \sum_{t\in T}\mathrm{SCC}_s\,\Delta^T_t\,\mathrm{p}^{\mathrm{INJ}}_{s,t,r}\Bigg)\Bigg] \tag{3.5}$$

subject to:

$$\sum_{\{b\in BC_i^N\,:\,\mathrm{month}_t\in M_b^{BC}\}} \mathrm{p}^{BC\,u}_{b,\mathrm{month}_t,r} + \sum_{\{o\in SC_i^N\,:\,t\in T_o^{OC}\}} \mathrm{p}^{OC}_{o,t,r} - \sum_{\{j\,:\,(i,j)\in L\}} \mathrm{p}^{L}_{i,j,t,r} + \sum_{\{j\,:\,(j,i)\in L\}} \mathrm{p}^{L}_{j,i,t,r} + \sum_{s\in S_i^N}(\mathrm{p}^{\mathrm{EXT}}_{s,t,r} - \mathrm{p}^{\mathrm{INJ}}_{s,t,r}) = \mathrm{LOAD}_{i,t,r},\quad \forall i\in N, t\in T, r\in R \tag{3.6}$$

and:

$$\mathrm{p}^{BC\,u}_{b,m,r} \leqslant \mathrm{p}^{BC\,p}_{b,m}, \quad \forall b \in BC,\ m \in M^{BC}_b,\ r \in R \tag{3.7}$$

$$\mathrm{p}^{OC}_{o,t,r} \leqslant \mathrm{pmax}^{OC}_{o,r}, \quad \forall o \in OC,\ t \in T^{OC}_o,\ r \in R \tag{3.8}$$

$$\mathrm{p}^{L}_{i,j,t,r} \leqslant \mathrm{pmax}^{L}_{i,j,r}, \quad \forall (i,j) \in L,\ t \in T,\ r \in R \tag{3.9}$$

$$\mathrm{p}^{L}_{i,j,t,r} \geqslant \mathrm{pmin}^{L}_{i,j,r}, \quad \forall (i,j) \in L,\ t \in T,\ r \in R \tag{3.10}$$

$$\mathrm{e}^{S}_{s,t,r} = \mathrm{e}^{S}_{s,t-1,r} + \Delta^T_t(\mathrm{p}^{\mathrm{INJ}}_{s,t,r} - \mathrm{p}^{\mathrm{EXT}}_{s,t,r}),\ \forall s \in S,\ t \in T \setminus \{1\},\ r \in R \tag{3.11}$$

$$\mathrm{e}^{S}_{s,1,r} = \mathrm{emax}^{S}_{s,r}, \quad \forall s \in S,\ r \in R \tag{3.12}$$

$$\mathrm{e}^{S}_{s,T,r} = \mathrm{emax}^{S}_{s,r}, \quad \forall s \in S,\ r \in R \tag{3.13}$$

$$\mathrm{e}^{S}_{s,t,r} \leqslant \mathrm{emax}^{S}_{s,r}, \quad \forall s \in S,\ t \in T,\ r \in R \tag{3.14}$$

$$\mathrm{p}^{\mathrm{EXT}}_{s,t,r} \leqslant \mathrm{pmax}^{\mathrm{EXT}}_{s,r}, \quad \forall s \in S,\ t \in T,\ r \in R \tag{3.15}$$

$$\mathrm{p}^{\mathrm{INJ}}_{s,t,r} \leqslant \mathrm{pmax}^{\mathrm{INJ}}_{s,r}, \quad \forall s \in S,\ t \in T,\ r \in R \tag{3.16}$$

$$0 \leqslant \mathrm{p}^{\mathrm{EXT}}_{s,t,r} \leqslant \mathrm{PMAX}^{\mathrm{EXT}}_{s,t}, \quad \forall s \in S,\ t \in T,\ r \in R \tag{3.17}$$

$$0 \leqslant \mathrm{p}^{\mathrm{INJ}}_{s,t,r} \leqslant \mathrm{PMAX}^{\mathrm{INJ}}_{s,t}, \quad \forall s \in S,\ t \in T,\ r \in R \tag{3.18}$$

$$0 \leqslant \mathrm{e}^{S}_{s,t,r} \leqslant \mathrm{EMAX}^{S}_{s}, \quad \forall s \in S,\ t \in T,\ r \in R \tag{3.19}$$

$$0 \leqslant \Delta^M_m \mathrm{p}^{BC\,p}_{b,m} \leqslant \mathrm{EMAX}^{BC}_{b,m}, \quad \forall b \in BC, m \in M^{BC}_b \tag{3.20}$$

$$0 \leqslant \sum_{t \in T^{OC}_o} \Delta^T_t \mathrm{p}^{OC}_{o,t,r} \leqslant \mathrm{EMAX}^{OC}_{o}, \quad \forall o \in OC,\ r \in R \tag{3.21}$$

$$0 \leqslant \mathrm{p}^{BC\,p}_{b,m} \leqslant \mathrm{PMAX}^{BC}_{b,m}, \quad \forall b \in BC,\ m \in M^{BC}_b \tag{3.22}$$

$$0 \leqslant \mathrm{p}^{OC}_{o,t,r} \leqslant \mathrm{PMAX}^{OC}_{o,t}, \quad \forall o \in OC,\ t \in T^{OC}_o,\ r \in R \tag{3.23}$$

$$\mathrm{p}^{BC\,u}_{b,m} \geqslant 0, \quad \forall b \in BC,\ m \in M^{BC}_b \tag{3.24}$$

$$\mathrm{pmin}^{L}_{i,j,r} \leqslant 0, \quad \mathrm{pmax}^{L}_{i,j,r} \geqslant 0, \quad \forall (i,j) \in L,\ r \in R \tag{3.25}$$

The complete model description is given in (3.1)–(3.25). The notation is summarized in Table 1 on the facing page. The objective function is stated as the difference of scenario-dependent revenues (3.2), purchase costs of baseload contracts (3.1) and open contracts (3.3) and costs of transportation (3.4) and storage (3.5). The balance of power levels as well as demand fulfillment is ensured by constraint set (3.6). Constraint set (3.7) distinguishes purchased and used baseload quantities. Peak purchase and transportation quantities are determined by constraint sets (3.8)–(3.10). Constraint sets (3.11)–(3.13) ensure initialization and balance of the storage fill levels. Peak storage fill levels and peak injection and extraction levels are determined by constraint sets (3.14)–(3.16). The remaining constraints (3.17)–(3.25) impose necessary bounds on the decision variables.

TABLE 1 Notation. [Table continues on next two pages.]

(a) Sets

Symbol	Definition
R	Set of scenarios
N	Set of nodes (virtual trading points, national entry points)
$L \subseteq N \times N$	Set of links
BC	Set of baseload contracts
OC	Set of open contracts
SC	Set of sales contracts
BC_i^N	Set of baseload contracts at node $i \in N$
OC_i^N	Set of open contracts at node $i \in N$
SC_i^N	Set of sales contracts at node $i \in N$
T	Set of time periods
M_b^{BC}	Set of months in which baseload contract $b \in BC$ is valid
T_o^{OC}	Set of time periods in which open contract $o \in OC$ is valid
S	Set of storage facilities
S_i^N	Set of storage facilities at node $i \in N$

As mentioned above, the accurate consideration of peaks in demand is crucial in order to represent costs accurately. In real-world natural gas trading, accounting is done on an hourly basis. Therefore, solving the model with $\Delta_t^T = 1$ (one time period equals one hour) results in the highest possible accuracy. Although the model is blown up to 8760 time periods given a planning horizon of one year, this variant turned out to be solvable for practical cases in the deterministic case (one scenario). For the stochastic case with many scenarios or inherent binary variables, for example, for storage fixed costs, a time aggregation had to be applied to achieve an acceptable solution characteristic. As pure averaging would distort or even eliminate short-term fluctuations, we resorted to representing a certain period of time (for example, a day) by three time periods of different lengths in the model: one for peak demand, one for medium demand and one for low demand. The parameter Δ_t^T is used to represent the length of an associated time period in hours and to enable the correct conversion from power units into energy units. Table 2 on page 60 gives an overview of typical time aggregations.

3.1 Incorporating conditional value-at-risk

Given a probability α, the CVaR is defined as the conditional mean value of the worst $(1 - \alpha) * 100\%$ of losses/profits. It was first proposed by Rockafellar and Uryasev

TABLE 1 Continued.

(b) Parameters

Symbol	Definition	Unit
ρ_r	Probability of scenario $r \in R$	
Δ_t^T, Δ_m^M	Length of time period $t \in T$/month m in hours	h
$\mathrm{LOAD}_{c,i,t,r}$	Gas demand from supply contract $c \in SC$ at node $i \in N$ in time period $t \in T$ and scenario $r \in R$	MW
$\mathrm{EPRICE}_{c,t}^{SC}$	Price of energy sold via open contract $c \in SC$ in time period $t \in T$	€/MWh
$\mathrm{EPRICE}_{b,m}^{BC}$	Price of energy purchased via baseload contract $b \in BC$ in month $m \in M_b^{BC}$	€/MWh
$\mathrm{EPRICE}_{o,t}^{OC}$	Price of energy purchased via open contract $o \in OC$ in time period $t \in T_o^{OC}$	€/MWh
PPRICE_{o}^{OC}	Price of maximum power level purchased via open contract $o \in OC$ during the planning horizon	€/MW
$\mathrm{PMAX}_{b,m}^{BC}$	Maximum power level to be purchased via baseload contract $b \in BC$ in month $m \in M_b^{BC}$	MW
$\mathrm{PMAX}_{o,t}^{OC}$	Maximum power level to be purchased via open contract $o \in OC$ in time period $t \in T_o^{OC}$	MW
$\mathrm{EMAX}_{b,m}^{BC}$	Maximum amount of energy to be purchased via baseload contract $b \in BC$ in month $m \in M_b^{BC}$	MWh
$\mathrm{EMAX}_{o,t}^{OC}$	Maximum amount of energy to be purchased via open contract $o \in OC$ in time period $t \in T_o^{OC}$	MWh
$\mathrm{CENTRY}_{i,j}$	Entry price of market region (node) $j \in N$	€/MW
$\mathrm{CEXIT}_{i,j}$	Exit price of market region (node) $i \in N$	€/MW
month_t	Index of month time period t belongs to	
$\mathrm{PPRICE}_s^{\mathrm{INJ}}$	Price for maximum power injected into storage $s \in S$	€/MW
$\mathrm{PPRICE}_s^{\mathrm{EXT}}$	Price for maximum power extracted from storage $s \in S$	€/MW
SCF_s	Fixed costs of storage $s \in S$	€
SCV_s	Variable storage costs, refer to maximum fill level of storage $s \in S$	€/MWh
SCC_s	Compression costs incurred by injecting one unit of energy into storage $s \in S$	€/MWh
EMAX_s^S	Maximum fill level of storage $s \in S$	MWh
$\mathrm{PMAX}_s^{\mathrm{INJ}}$	Maximum injection power of storage $s \in S$ in time period $t \in T$	MW
$\mathrm{PMAX}_s^{\mathrm{EXT}}$	Maximum extraction power of storage $s \in S$ in time period $t \in T$	MW

TABLE 1 Continued.

(c) Variables

Symbol	Definition	Unit
$\mathrm{p}_{b,m}^{BC\ p}$	Amount of power purchased via baseload contract $b \in BC$ in month $m \in M_b^{BC}$	MW
$\mathrm{p}_{b,m}^{BC\ u}$	Amount of power used from baseload contract $b \in BC$ in month $m \in M_b^{BC}$	MW
$\mathrm{p}_{o,t}^{OC}$	Amount of power purchased via open contract $o \in OC$ in time period $t \in T_o^{OC}$	MW
pmax_o^{OC}	Maximum power level purchase via open contract $o \in OC$	MW
$\mathrm{p}_{i,j,t}^{L}$	Power level on link $(i, j) \in L$ in time period $t \in T$ (can be negative)	MW
$\mathrm{pmin}_{i,j}^{L}$	Minimal power level on link $(i, j) \in L$ in time period $t \in T$	MW
$\mathrm{pmax}_{i,j}^{L}$	Maximum power level on link $(i, j) \in L$ in time period $t \in T$	MW
$\mathrm{p}_{s,t}^{\mathrm{INJ}}$	Amount of power injected into storage $s \in S$ in time period $t \in T$	MW
$\mathrm{p}_{s,t}^{\mathrm{EXT}}$	Amount of power extracted from storage $s \in S$ in time period $t \in T$	MW
$\mathrm{pmax}_s^{\mathrm{INJ}}$	Maximum used injection power at storage $s \in S$	MW
$\mathrm{pmax}_s^{\mathrm{EXT}}$	Maximum used extraction power at storage $s \in S$	MW
$\mathrm{e}_{s,t}^{S}$	Amount of energy stored in storage s (fill level) in time period $t \in T$	MWh
emax_s^{S}	Maximum amount of energy stored in storage $s \in S$	MWh
emin_s^{S}	Minimum amount of energy stored in storage $s \in S$	MWh

(2002). The authors also show its computational tractability by representing CVaR as the optimum of a special minimization problem. It is well-known that, in the case of discrete finite distributions, CVaR optimization problems can be formulated as linear programming problems. To integrate CVaR into our model, we use the following formulation of Fábián (2008):

$$y_0 + y_r \leqslant Q_r/(1-\alpha) \quad \forall r \in R \tag{3.26}$$

$$y_r \geqslant 0 \quad \forall r \in R \tag{3.27}$$

TABLE 2 Hourly, daily and weekly time aggregation.

	Length of one time period (hours)	Number of time periods
	Hourly	
	$\Delta_t^T = 1 \ \forall t \in T$	8760
	Daily	
	$\Delta_t^T = 3$ if $(t \bmod 3) + 1 = 0$	1095
	$\Delta_t^T = 13$ if $(t \bmod 3) + 2 = 0$	
	$\Delta_t^T = 8$ if $(t \bmod 3) = 0$	
	Weekly	
	$\Delta_t^T = 20$ if $(t \bmod 3) + 1 = 0$	159
	$\Delta_t^T = 93$ if $(t \bmod 3) + 2 = 0$	
	$\Delta_t^T = 55$ if $(t \bmod 3) = 0$	

where y_0 and y_r denote $|R| + 1$ additional continuous decision variables (y_0 is generally unbounded), and Q_r denotes the stage-two objective function value of the rth scenario, which in our case is equal to the term in square brackets in Equations (3.2)–(3.5). The variable y_0 will assume the CVaR and can hence be used either to optimize or limit it from below. In our case, the additional computational burden of this formulation seems to be negligible given our relatively large core model.

4 CASE STUDY

4.1 Description of case study

Our case study is based on a real-world planning situation at a large German public utility. The underlying network consists of eleven nodes, ten arcs and two storage facilities. Three of the nodes represent sales regions with their respective uninterruptible sales contracts. The gas demand can be satisfied via purchase from two monthly baseload contracts and one open contract. We also experimented with including a spot market that is largely modeled as a storage.

For the three sales regions, real-world hourly load data was available for three former planning years. Furthermore, we obtained temperature data consisting of daily temperature averages for one hundred years. In order to generate hourly load scenarios, the dependency between average daily gas demand and average daily temperature was approximated by a piecewise linear function based on the real-world load data of one specific year. This piecewise linear function was then used to generate daily load

TABLE 3 Impact of time aggregation on model size and solution time.

Time aggregation	Time periods	Vars	Rows	Nonzeros	MOPS IPM 32 CPU time (seconds)
Weekly	159	5 572	13 579	33 926	1
Daily	1095	32 716	90 331	227 678	13
Hourly	8760	255 001	718 861	1 814 333	398

averages for a set of daily temperature data of a certain year. Hourly load scenarios were obtained by starting from the artificially generated daily load data and mimicking the daily load fluctuations of the corresponding reference load data in a proportional way. In the model variant that includes a spot market, spot-market prices were assumed to behave in a largely temperature-dependent way and were adjusted accordingly using a simple proportional scheme.

This method of scenario generation was chosen over representing load scenarios by a stochastic model for two reasons. Firstly, the simplicity and conceivability of the scenario generation procedure fostered the acceptance of the whole approach by the companies purchase portfolio planner. Secondly, it is crucial to take into account the specific regional demand characteristic, such as peak demand on certain days of the year, in a transparent way. These characteristics are highly dependent on the regional customer portfolio and can hardly be represented adequately by a general stochastic model.

4.2 Computational experiences

Table 3 shows computational results for the deterministic model (one scenario) and different time aggregation. The problem instances were solved using the interior point code of the solver MOPS 9.29 (see Suhl (2008)) on a standard 32-bit INTEL PENTIUM IV PC with 3.2GHz and 2GB main memory. Since no basic solution is required in this case, the crossover method was switched off to speed up the solution process. Clearly, time aggregation has a major impact on model sizes and solution times, but, for the deterministic case, all variants were well solvable. In our experiments, simplex-type solvers turned out to be highly inferior to interior point solvers. However, in some cases, the dual simplex algorithm was numerically more stable. Furthermore, simplex solvers typically use much less memory than interior point solvers. A comparison of objective function values resulted in a deviation of 0.2% for the daily time aggregation and 3.2% for the weekly time aggregation with respect to the hourly time aggregation.

Table 4 on the next page shows some computational results for the stochastic model. The deterministic equivalent was solved on an INTEL Core 2 Duo 2.2GHz and

TABLE 4 Computational results for the stochastic model (deterministic equivalent).

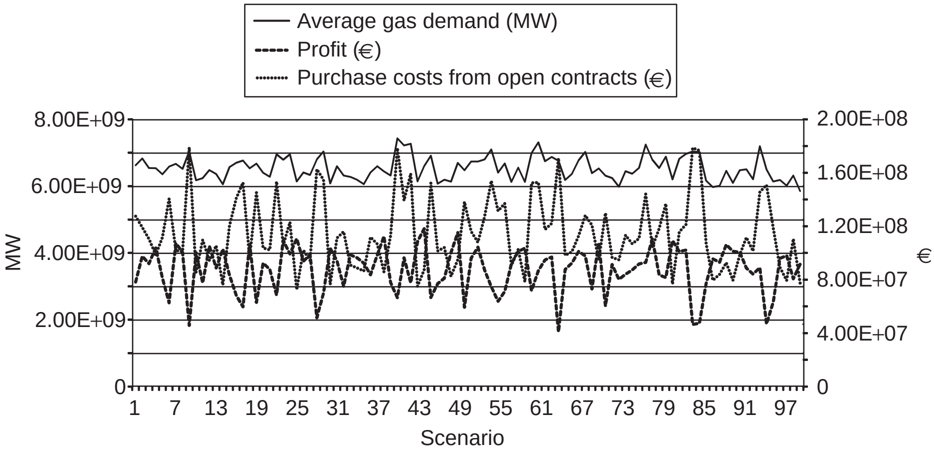
Time aggregation	Scenarios	Rows	Vars	Nonzeros	MOPS IPM 64 CPU time (seconds)	Cplex 11.0 barrier CPU time (seconds)
Weekly	500	1 958 061	4 341 028	11 369 098	475	4702
Daily	50	1 527 438	3 774 246	9 682 258	1995	Failed
Hourly	6	1 468 265	3 787 374	9 624 410	398	Failed

8GB of main memory using the 64-bit versions of the solvers MOPS IPM 9.29 and ILOG Cplex 11.0 Barrier (ILOG (2008)). It can be seen that the stochastic model can be solved for a considerable number of scenarios with weekly time aggregation only. Further computational tests conducted by Zverovich *et al* (2009) indicate that using state-of-the-art decomposition methods does not outperform the deterministic equivalent using an interior point solver on our model instances. As in our results, the Cplex solver failed to solve most of the problem instances due to numerical difficulties. This confirms our experience that the problem instances generated from our model are often numerically challenging.

4.3 Impact of uncertainty and benefit of using the stochastic model

In this section we discuss the impact of demand uncertainty and the benefit of using a stochastic instead of a deterministic model. For this purpose we will make use of the well-known decision approaches for planning situations under uncertainty, namely the expected-value (EV), wait-and-see (WS) and here-and-now (HN) approaches, as well as the stochastic measures expected value of perfect information (EVPI) and value of the stochastic solution (VSS) (see Birge and Louveaux (1997) for definitions). Figure 3 on the facing page displays average gas demand together with profits and total purchase costs from open contracts resulting from the use of the stage-one solution of the stochastic model (HN solution). Obviously, in our case study, the main risk of the portfolio planner is lack of supply from baseload contracts in years of peak demand. In these scenarios, high costs from open-purchase contracts result in low profits (for example, in scenarios 9 and 40). However, low profits can also occur in a year of medium demand as in scenarios 64 and 95. Closer analysis reveals that, in these years, peaks in demand occur in late spring and happen to match with low storage fill levels. On the other hand, warm winters can lead to a surplus of gas from baseload contracts. This situation was obviously avoided by the solution of the stochastic model.

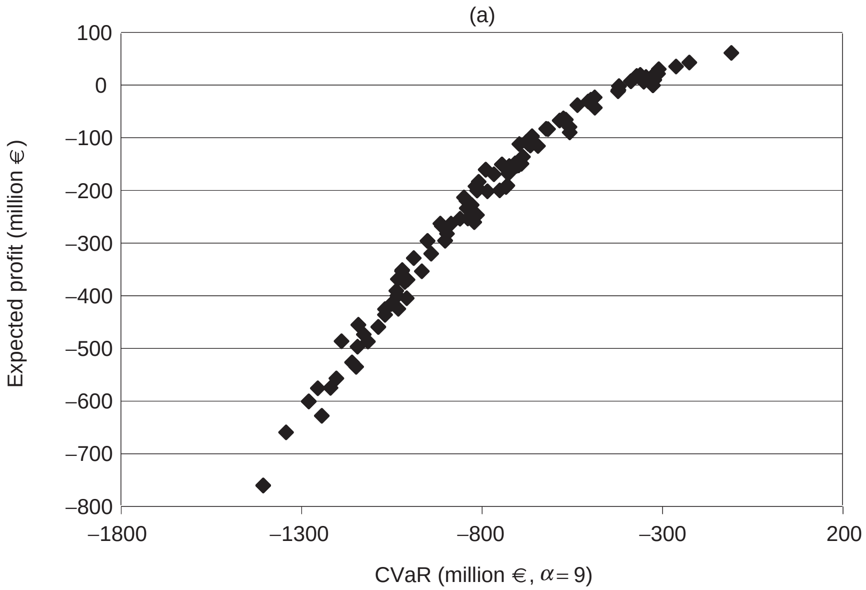
FIGURE 3 Average gas demand, profits and open-purchase costs of the HN solution for one hundred scenarios.

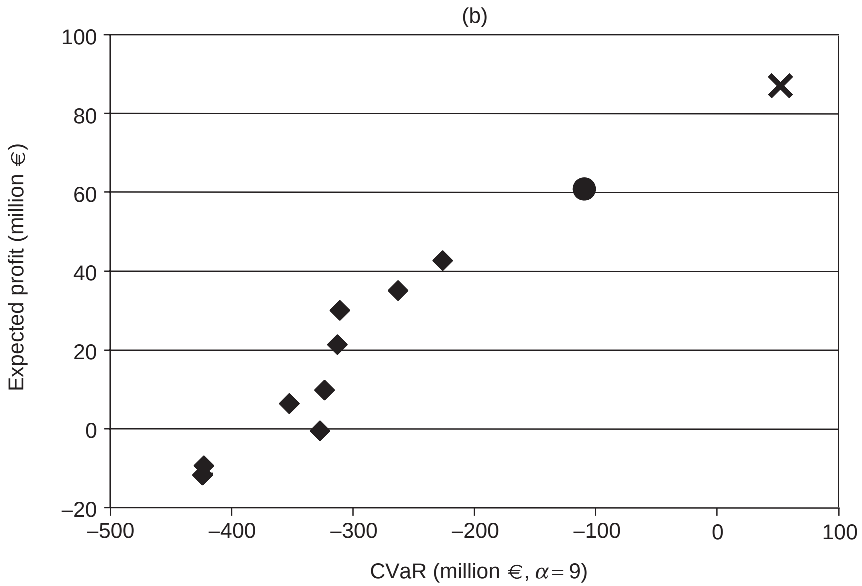


Given the high volatility of demand and the high impact of peak demand on costs, it is unlikely that the decision maker would ever use an EV approach to determine what baseload contracts to engage in. By using the EV approach, the demand curve becomes smoothed, and as such this leads to either overestimation or underestimation. Consequently, the decision maker will incur large expenses by often having to buy gas in the open contracts market. However, for clarity in illustrating the various stochastic measures, we present the full set of stochastic measures, some of which are a consequence of comparison to the EV approach. As can be deduced from Table 5 on page 65, the impact of uncertainty EVPI is dramatic, particularly if no use is made of the spot market. If stage-one decisions cannot be partly corrected by purchasing from and selling to the spot market, the EV solution behaves catastrophically for the reasons discussed above. The solution of the stochastic model does very well in both configurations, which is confirmed by a high VSS. It has to be remarked that the results that include the spot market have to be interpreted with care. As mentioned above, the modeling of the spot market is only rudimentary – for example, full uncertainty of spot prices is hardly taken into account. However, we think that Table 5 on page 65 gives a largely correct impression of the benefits of using the stochastic model.

In order to make a more realistic comparison with what the decision maker may do in reality, we performed a scenario analysis, as shown in Figure 4 on the next page. In this analysis, an optimal stage-one solution of a certain scenario is evaluated in all of the remaining scenarios by fixing the stage-one variables accordingly. The associated expected profit and risk values in terms of CVaR are depicted in part (a)

FIGURE 4 Scenario analysis.





Part (a) displays expected profits and CVaR associated with stage-one planning solutions generated by optimizing each scenario separately. Part (b) displays the stage-one solutions associated with the ten highest load scenarios only, amended by the solution of the stochastic model (marked by the cross).

TABLE 5 Stochastic measures (millions of euros).

	With spot market	Without spot market
EEV	176.3	–193.4
WS	185.3	104.1
HN	182.2	87.3
EVPI	3.1	16.9
VSS	5.9	280.7

EEV: expected value of using the expected-value solution. WS: wait-and-see value. HN: here-and-now value (the optimal solution value of the stochastic model). EVPI: expected value of perfect information. VSS: value of the stochastic solution.

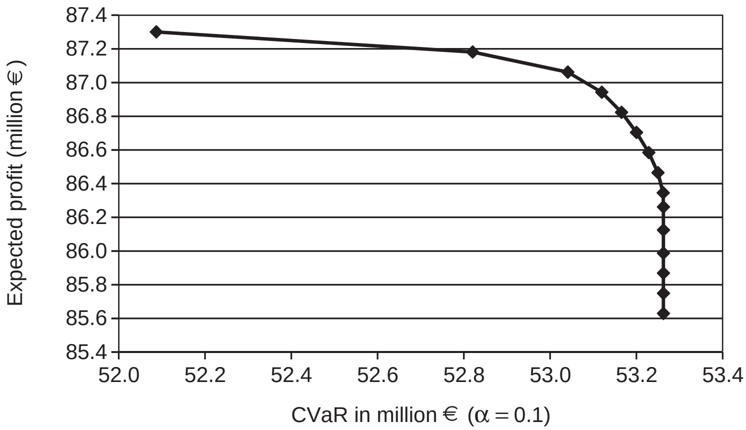
of Figure 4 on the facing page for all of the one hundred scenarios. It is clear to see that the vast majority of the scenarios lead to stage-one solutions associated with catastrophic negative expected profit and risk values. In fact, none of the stage-one solutions reaches a positive CVaR. Since the decision maker would typically conduct a scenario analysis looking at the colder years only, we display the ten scenarios with the highest average gas demand separately in part (b) of Figure 4 on the facing page. In this case, the coldest planning scenario, which is marked by the dot, yields the best stage-one solution, however, this still has a negative CVaR. Note that two of the solutions even yield a negative expected profit. The diagram is amended by the solution of the stochastic model marked by the cross. Note that it clearly outperforms the solutions of the coldest year in terms of expected profit. Furthermore, it is the only stage-one solution which reaches a clearly positive CVaR.

To summarize these results, we have found that, although the deterministic approach described above is much better than the EV approach, there is still too much volatility in the demand to ensure a robust solution. The stochastic programming solution clearly seems the best method for hedging against the volatility of demand and the consequential purchasing of gas on the open contracts market.

4.4 Explicit risk optimization

Figure 5 on the next page displays the efficient frontier with respect to maximizing expected profit and CVaR. It was constructed by subsequent optimization runs with growing lower limits on profit while maximizing CVaR. While the graph clearly shows the trade-off between risk and profit, the absolute gains in robustness that come from sacrificing expected profit are small. We also experimented with adding a CVaR constraint to the model to restrict the tail distribution of profit. Interestingly, this has a very limited impact, ie, the HN solution already possesses a very good risk characteristic in terms of CVaR.

FIGURE 5 Efficient risk–return frontier.



4.5 Lessons learned from the case study

In solving this problem by investigating all the classical stochastic programming measures of quality, and conducting a scenario analysis, it is clearly evident that the proposed purchase strategy of the two-stage stochastic programming problem provides the decision maker with good guidance compared with the EV and "cold year" approaches. By taking on board the proposed purchase strategies the problem owner will be able to make a more informed analysis on these decisions by evaluating them against the problem owner's perceived important scenarios. We have found that the critical issue with regard to risk is the gas that has to be purchased on the open contracts. Our analyses have illustrated that the two-stage strategy has performed well in the extreme situations of cold winters or cold late springs. In the situation of warm winters the results also perform reasonably well, in contrast to the best achievable. Limiting risk explicitly by adding a CVaR constraint did not have a large impact on the optimal expected profit. However, our main aim in this investigation was to illustrate the benefit for the current model of accounting for uncertainty in demand and using two-stage stochastic programming to propose a gas purchasing strategy. This investigation clearly illustrates this benefit over a deterministic approach.

5 CONCLUSIONS

We presented a two-stage stochastic linear programming model for the strategic gas-purchase problem faced by local distribution companies. We enhanced the model to explicitly consider the CVaR and evaluated our approach based on a real-world case study. The results show that our model is computationally tractable using a standard interior point solver for hundreds of scenarios. A scenario analysis showed that the purchase portfolios generated by the stochastic model outperform those from deterministic approaches in terms of both expected profit and robustness. Several aspects require further research, such as improved consideration of price uncertainty, integration of the futures market, modeling of quantity discounts and the application of the model in an operational, rolling-horizon-type planning situation.

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